Eur. Phys. J. B **50**, 659–669 (2006)
DOI: 10.1140/epjb/e2006-00169-x

**The European Physical Journal B**

# Modeling the net flows of U.S. mutual funds with stochastic catastrophe theory

A. Clark[a]

Lipper, a Reuters Company, 707 17th Street, 22nd Floor, Denver, CO 80202, USA

Received 7 June 2005 / Received in final form 25 November 2005
Published online 17 May 2006 – 

**Abstract.** Using the recent work of Hartelman, van der Maas, and Wagenmakers, we demonstrate the use of invariant stochastic catastrophe models in finance for modeling net flows (the difference between purchases and redemptions of fund shares) of U.S. mutual funds. We validate Goetzmann et al. and others' work concerning the importance of sentiment variables on stock fund flows. We also answer some of the questions Goetzmann et al. and Brown et al. pose at the end of their respective papers. We end with possible experiments for experimental economists and sociophysicists.

**PACS.** 89.65.Gh Economics, econophysics, financial markets, business and management – 87.23.Ge Dynamics of social systems – 05.10.-a Computational methods in statistical physics and nonlinear dynamics

## 1 Introduction

During the past ten years there has been a growing interest in the hows and whys of net flows of U.S. mutual funds (the difference between purchases of fund shares and redemptions of fund shares). Many of these studies ([1–3] are a small sample) have focused on the determinants of flows into individual mutual funds or groups of mutual funds. Most of these studies have found repeatedly the importance of prior-period returns on the next period's flows [1,2] and the importance of the rankings of risk-adjusted returns from prior period(s) on the current period's flows [3].

There have been relatively few papers other than those of Goetzmann et al. [4] and Brown et al. [5] that have looked at fund flows from a broader perspective, e.g., all stock fund flows, all bond fund flows, all money market fund flows, all gold or precious metals fund flows, and all bear fund (a fund whose price rises when the stock market falls) flows.

Goetzmann et al. and Brown et al. took this broader approach, since they were looking for a priceable sentiment variable as hypothesized by de Long et al. [6]. Both Goetzmann et al. and Brown et al. found that sentiment or behavioral variables had a significant impact on stock fund flows and that more than likely "a pervasive investor sentiment variable" does exist [4], but they found no solid proof of a priceable sentiment variable [5].

In this paper we build on the work of Goetzmann et al. and Brown et al. by including their sentiment variables, as well as other variables, in our study of stock mutual fund flows by using an invariant stochastic catastrophe model that was developed by Hartelman [7] and tested by Ploeger et al. [8], van der Maas et al. [9], and Wagenmakers et al. [10,11].

We find the cusp catastrophe model provides a fairly simple and elegant model of investor behavior on both a daily and monthly basis. And while we have evidence that a butterfly model may better describe the daily data, the results for the monthly data are inconclusive, pointing to the need for more tests.

The monthly model relies on the prior month's returns of the Dow Jones Industrial Average (DJIA) as the normal variable, with the bias variables being the monthly down volume of the New York Stock Exchange (NYSE) and the monthly net flows of money market mutual funds.

Our interpretation of this model is that U.S. mutual fund investors (about 20% of the U.S. stock investor universe) tend to behave no differently than the remaining 80% of the U.S. stock investor universe. By this we mean that mutual fund investors follow stock price movements (the normal variable) as they make their rebalancing and investment decisions, while the sentiment/splitting variables (the down volume on the NYSE and flows in or out of money market accounts) measure an investor's willingness to commit money to the stock market versus the bond or other markets. This willingness may arise, as both Goetzmann et al. and Brown et al. have noted, because of views about the future equity premium or because the investor is following a common portfolio insurance strategy.

In our daily model we fit a cusp model similar to the monthly model above, i.e., lagged stock returns on the normal side and sentiment variables on the splitting side. In this case the splitting variable does not include a volume variable but does include the net flows of bear funds (funds that benefit when stock prices fall).

[a] e-mail: andrew.clark@lipper.reuters.com

From our findings we think, like Goetzmann et al. and Brown et al., that there is a priceable sentiment variable, but that it is nonlinear in nature and more than likely stochastic — as Goetzmann et al. mention in their paper. Stated in a stronger manner, we think looking for a priceable sentiment variable using a linear version of APT-as Goetzmann et al. do-or a linear factor model-as Brown et al. do-will not succeed because of the nonlinearity of the data. We favor an interpretation similar to that of Shefrin [12]: the pricing kernel exists in two pieces-one term being sentiment and the other being an expression that depends on economic fundamentals. We think the sentiment term Shefrin describes, with its bimodality, bears a strong resemblance to the features we find in U.S. mutual fund flows and their multimodality.

In Section 2 we review the data used. Section 3 reviews both Cobb et al.'s [13–17] and Hartelman's [7] work on stochastic catastrophe theory (SCT), while Section 4 goes over the results of our models. Section 5 contains our conclusions.

## 2 Data used

The variables we use in this paper are: monthly and daily net flows of U.S. stock mutual funds, monthly and daily returns of the DJIA, monthly and daily net flows of money market mutual fund accounts, monthly and daily volume of the NYSE (in total and in its two pieces: up volume and down volume), monthly retail sales data, daily gold fund net flows, daily and monthly bond fund net flows, daily net flows of international stock funds, and daily net flows of bear funds.

We use this particular set of variables for several different reasons. First, we had already built, prior to the work in this paper, a one-month-ahead forecast model of stock fund net flows that uses all the monthly variables above, with the exception of the bond fund data and the up and down volumes of the NYSE.

Secondly, Goetzmann et al. and Brown et al. have found all of the daily variables listed above (with the exception of the NYSE volume variables) helpful in explaining the net flows of stock funds. And, a number of studies such as Walther [1] and Del Guercio and Tkac [3] have demonstrated the importance of securities returns on fund flows.

Our variable list, then, covers the most common factors we and others have used to explain stock fund flows, in particular the three sentiment variables (money market account flows, gold fund flows, and bear fund flows) that Goetzmann et al. and Brown et al. have found significant.

As mentioned above, our monthly data cover the period November 1984 through August 2005, and our daily data go from April 3, 2000, through December 30, 2004. The sources of our monthly data include the Investment Company Institute (ICI) for monthly flows of stock, bond, and money market funds and Reuters for retail sales data, the month-end prices of the DJIA, and the NYSE volume figures.

The sources of our daily data are Lipper[1] for all fund flows data and Reuters for daily prices of the DJIA and the NYSE volume figures.

For the monthly data, since we are working with more than 20 years of observations, we inflation-adjusted the flows, NYSE volume, and retail sales by using Bureau of Labor Statistics methods, so we could state everything in current (2005) dollars.

Since inflation has been relatively low over the period of our daily model, we did not inflation-adjust any variables there.

Only three of the monthly variables — money market fund flows, DJIA returns, and bond fund flows (the first two are show in Fig. 1) — are stationary as can be confirmed from visual inspection. We also confirmed the stationarity of these series via the Kwiatkowski-Phillips-Schmidt-Shin (KPSS)[2] and the Leybourne and McCabe (LMC)[3] tests. The remaining monthly variables (two of which are shown in Fig. 2) are not stationary.

To remove the trend in these series we fit a Henderson trend-cycle[4] to the data and worked with the residuals from our model fitting. Figure 3 shows the effectiveness of the Henderson fit, i.e., both the U.S. stock fund flows and the retail sales now appear stationary.

We also confirmed the results of our transformation by using both the KPSS and LMC tests, finding all the transformed data had indeed become stationary.

The importance of stationarity to our cusp model was made clear by a referee's comment. The referee wrote that nonstationarity can lead to spurious detection of catastrophe models, so stationarity is necessary to properly interpret the results of our tests and fits.

For the daily data there was no need to fit the Henderson model, since our tests for stationarity via KPSS and LMC revealed all the daily variables were indeed stationary.

[1] Lipper, a wholly owned subsidiary of Reuters, collects and analyzes data on more than 100 000 mutual funds world-wide and through its sister firm — Hedge World — collects data on more than 6000 hedge funds globally.

[2] The KPSS test [18] is a Lagrange multiplier test for the null hypothesis that the error variance in the random walk component of the series is zero. The version we used is in the R package *timeseriesTests*.

[3] The LMC test [19] is based on the same model as the KPSS test, with its difference being how the nonparametric estimator of the long-run variance is computed. The test was written by the author using the R language.

[4] The Henderson trend-cycle as defined in Statistica® is: "The X-11 [census] seasonally adjusted series is smoothed via a variable moving average procedure. In general, the so-called Henderson curve moving average is applied, which is a weighted moving average with the magnitudes of the weights following a bell-shaped curve." The reference given by Statistica® for the Henderson methodology is Shiskin, J., Young, A.H., and Musgrave, J.C. (1967): "The X-11 Variant of Census Method II Seasonal Adjustment," *Technical Paper No. 15, Bureau of the Census*, U.S. Department of Commerce.

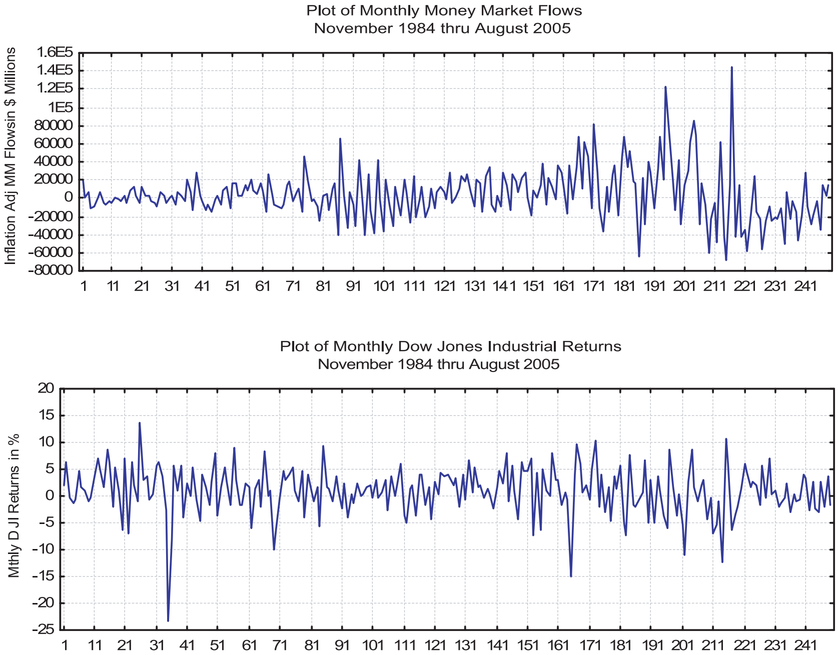


**Fig. 1.** Plot of monthly money market mutual fund net flows and monthly dow jones industrial average returns.

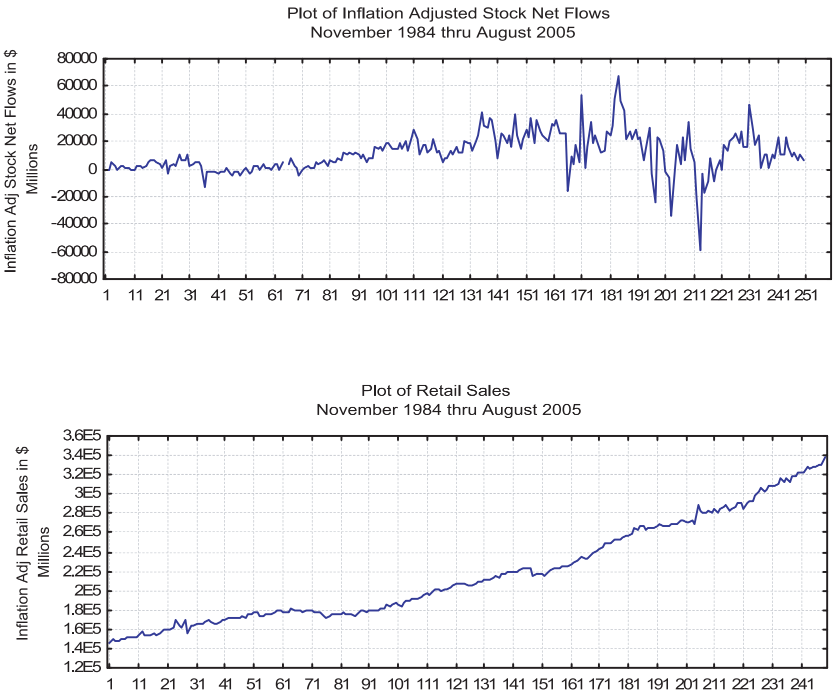


**Fig. 2.** Plot of U.S. stock fund net flows and retail sales.

## 3 Catastrophe models

Since the initial work of Thom [20], Zeeman [21], and others [22,23], catastrophe theory has found a home in physics and chemistry but has been basically ignored by economists until recently for a number of reasons. There have been, however, a few prominent exceptions. One is a paper by Zeeman [24], which was roundly criticized when it first appeared, and the other is a more-recent and very interesting paper by Rosser [25], in which the first application of the butterfly catastrophe model to stock markets was proposed.

One of the main reasons, according to Rosser [26], that catastrophe theory has had such a tough time being accepted in the economics community is that papers such as those written by Zahler and Sussmann [27] and Weintraub [28] found serious "flaws" in the use of catastrophe theory in the social sciences. A recent critique of Zahler and Sussmann and Weintraub by Rosser [26] answers

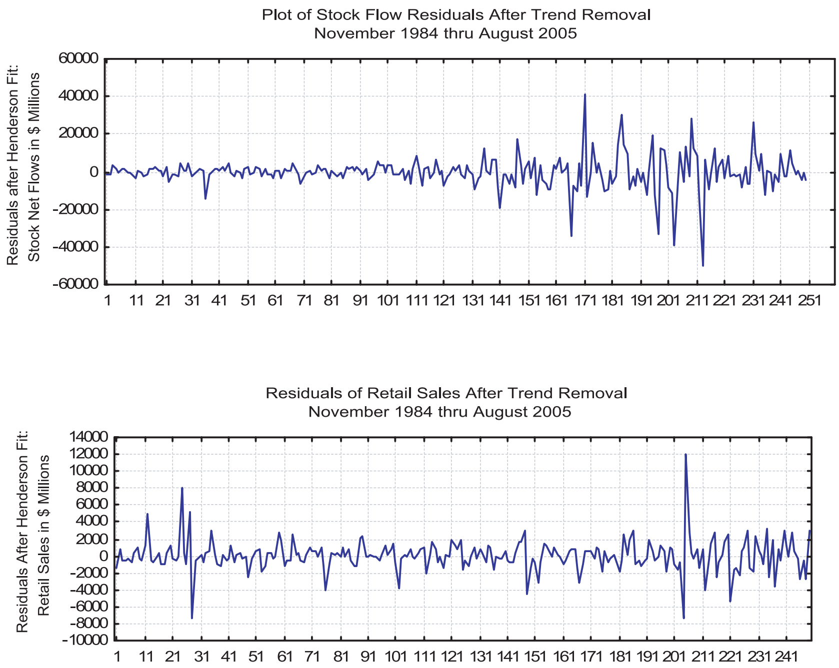


**Fig. 3.** Plot of detrended monthly U.S. stock fund net flows and retail sales.

several of these concerns and makes a very good case that, as Rosser puts it, "the baby was thrown out with the bath water" when it came to catastrophe theory.

As Rosser mentions in his paper, Cobb et al.'s work on SCT [13–17] was a major step forward in answering some of the criticisms put forth in Zahler and Sussmann's paper.

Describing Cobb's work, Wagenmakers et al. [11] write, "the deterministic behavior (*of a single state variable catastrophe model*) can be made stochastic and put in the form of a stochastic differential equation (SDE) by simply adding a stochastic Gaussian white noise term dW(t):

$$dx/dt = -dV\,(x)\,/dx + \sigma\,(x)\,dW\,(t)\,/dt. \qquad (1)$$

The deterministic term on the right-hand side, $-dV(x)/dx$, is the drift term $\mu(x)$, while $\sigma(x)$ is the diffusion function, and $W(t)$ is a Wiener process".

Paraphrasing some further comments by Wagenmakers et al. on Cobb's work, under the assumption of additive noise, i.e., $\sigma\,(x)$ is constant and depends only on $x$, it can be shown the modes or local maxima of the empirical pdf correspond to stable equilibrium (Honerkamp [29]). More generally, there is a simple one-to-one correspondence between the additive noise SDE and its pdf. Instead of fitting the drift function of the cusp model directly, one can fit the pdf.

$$p(y|\alpha,\beta) = N \exp\left[-\frac{1}{4}y^4 + \frac{1}{2}\beta y^2 + \alpha y\right] \qquad (2)$$

where $N$ is a normalizing constant. The dependent variable $x$ in equation (1) has been rescaled in equation (2) by $y = (x-\lambda)/\sigma$, and $\alpha$ and $\beta$ are linear functions (the normal and splitting variables, respectively). The two control variables $a$ and $b$ enter the equation by $\alpha = k_0 + k_1 a + k_2 b$ and $\beta = l_0 + l_1 a + l_2 b$. The parameters $\lambda, \sigma, k_0, k_1, k_2, l_0, l_1$, and $l_2$ can be estimated using MLE (Cobb and Watson [30]).

The problem with the above technique, as noted by Hartelman [7], is that it can lead to an excess of model parameters. As Hartelman writes, "Therefore, every dataset can be fitted perfectly to a catastrophe model." This problem exists because in practical applications of catastrophe theory we have only limited information about the process, and it is impossible to reconstruct from this information the complete potential function and its higher derivatives without making some assumptions.

Hartelman solved this problem by smoothing the data *a la* Silverman [31]. The pdf has to be smooth for catastrophe theory to be applied unrestrictedly.

Hartelman uses Parzen's [32] smoother:

$$\widehat{f}_n\,(y) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{h_n}K\left(\frac{y-Y_i}{h_n}\right) \qquad (3)$$

where $Y_i$ $i = 1, \ldots, n$ are the observations.

The kernel density estimate — equation (3) — can be easily generalized to multiple dimensions, and the window solutions Hartelman discusses also carry over easily to multi-dimensional cases.

Now, it is obvious the kernel estimate is smooth if the kernel function $K$ is smooth. Contrary to the parametric approach of Cobb, Invariant Stochastic Catastrophe Theory (ISCT) can be applied unrestrictedly to the kernel estimate. This means we can locate degenerate critical points; determine the Hessian matrix; apply the splitting lemma; look for symmetries; and ultimately, in the case of the occurrence of a degenerate critical point, determine

**Table 1.** Descriptive statistics of monthly inflation-adjusted and detrended U.S. stock fund flows (November 1984 through August 2005 (in millions of U.S. dollars)) and daily U.S. stock fund flows (April 3, 2000, through December 30, 2004).

| Descriptive Statistics | Monthly Stock Fund Net Flows ($ Millions) | Daily Stock Fund Net Flows ($ Millions) |
|---|---|---|
| No. of Observations | 250 | 1,140 |
| Mean | 8,493.15 | –11.9 |
| Median | 6,953.0 | –29.1 |
| Mode | Multiple | Multiple |
| Minimum | –50,122.5 | –18,307.2 |
| Maximum | 41,139.8 | 13,387.4 |
| Lower Quartile | –1,166.8 | –1,134.3 |
| Upper Quartile | 14930.0 | 1,013.9 |
| Standard Deviation | 13,272.7 | 2,231.1 |
| Skewness | –0.46 | 3.08 |
| Std. Error of Skewness | 0.26 | 0.08 |
| Kurtosis | 3.44 | 56.57 |
| Std. Error of Kurtosis | 0.51 | 0.16 |

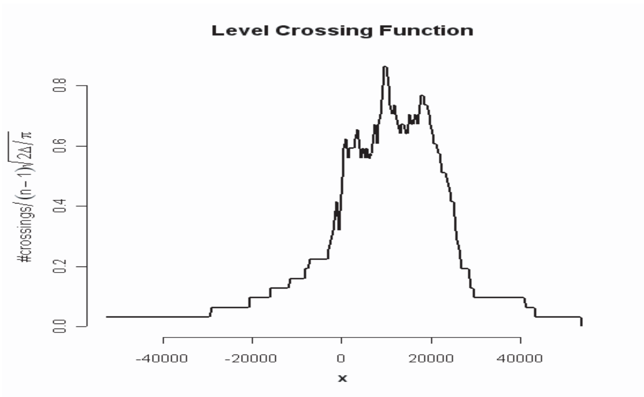


**Fig. 4.** Level crossing function of inflation-adjusted and detrended monthly U.S. stock fund flows.

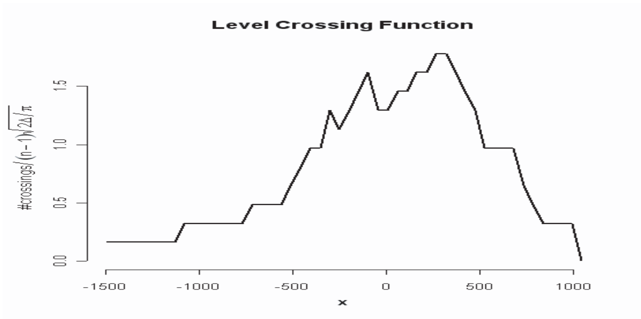


**Fig. 5.** Level crossing function of daily U.S. stock fund net flows.

the diffeomorphic transformations that cast the function locally into a canonical catastrophe function.

Hartelman has bundled most of these tools into three applications, available at Han van der Maas's Web site (`http://users.fmg.uva.nl/hvandermaas/`).

Taking advantage of Hartelman's tools, we examine our flows data — 250 months of U.S. stock fund net flows and almost five years of daily data. Table 1 details the descriptive statistics of stock fund flows data on both a monthly (inflation-adjusted and detrended) basis and on a daily basis.

Our first test is for multimodality, in particular bimodality-one of Gilmore's eight catastrophe flags [22]. We use Hartelman's level crossing program instead of a histogram to look for multimodality, since as noted in Wagenmakers et al. [11] there can be an inconsistency between the pdf and the invariant function with respect to the number of stable states. Examples of these inconsistencies are given in Wagenmakers et al. [11]. The monthly net flows data we use here are the residuals from the Henderson fit to the monthly U.S. stock fund flows data in Figure 4 and the unadjusted daily stock fund flows data in Figure 5.

Our net flows data are clearly multimodal, with two fairly distinct peaks in each case.

Our second test or flag is to look for sudden jumps in the flows data. Figure 3 above shows sudden jumps in the monthly U.S. stock fund flows series, while Figure 6 shows the same for the daily data.

From both Figures 3 and 6 it can be seen that neither stock function is continuous — and at least is potentially nonlinear. And, while we think this demonstration of such potentially discontinuous or nonlinear function can be an important indicator of catastrophe behavior, we also know that continuous acceleration models such as higher-order polynomials and logistic functions can fit a strictly discontinuous function as well.

In a third test we make use of Hartelman's [7] demonstration that kernel density estimates[5] can to be used to test for bifurcations in the data. We apply Hartelman's kernel program to test for bifurcations/degeneracies and find there is a probability of 0.80 to 0.90 that the flows data have at least one bifurcation on the monthly side and 0.75 or higher on the daily side.

Finally, we look at three other catastrophe flags — anomalous variance, critical slowing, and divergence of

[5] The kernel program is available at
`http://users.fmg.uva.nl/hvandermaas/`

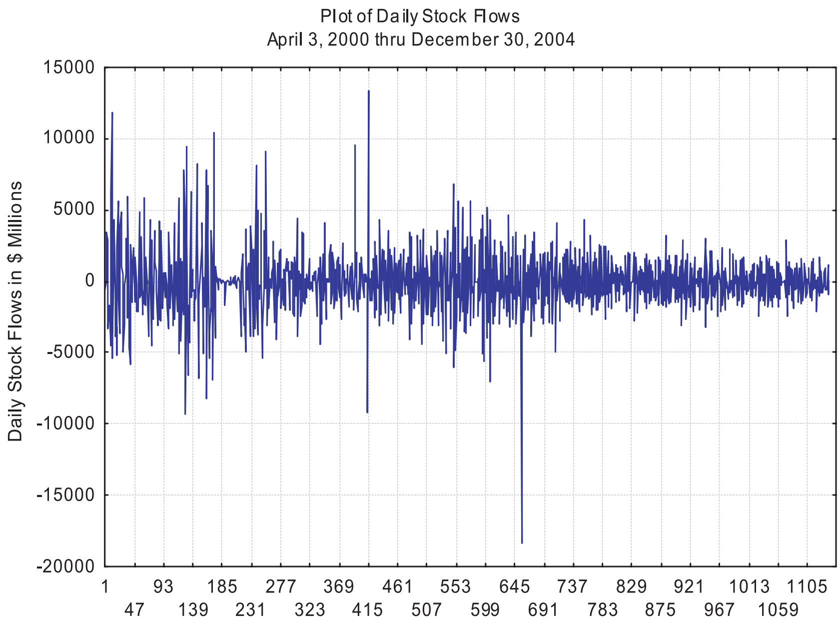


**Fig. 6.** Time series plot of daily U.S. stock fund net flows data.

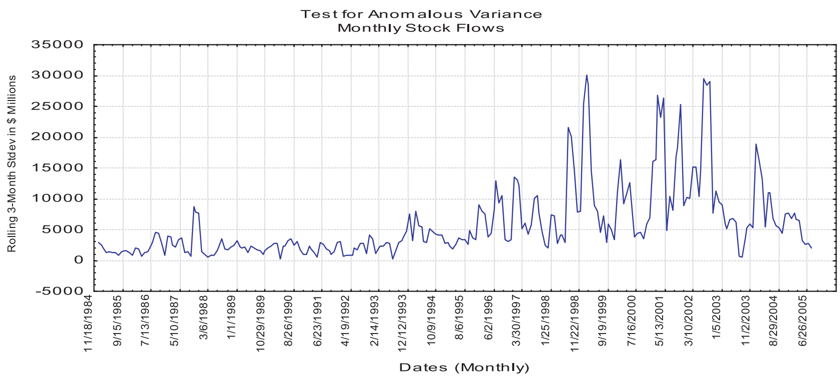


**Fig. 7.** Test for anomalous variance in monthly U.S. stock fund flows data.

linear response — to see if our flows data exhibit these qualities as well. In general, anomalous variance means that variance changes markedly near the transition points. In Figures 7 (monthly data) and 8 (daily data) we plot three-month rolling standard deviations of both stock series over time.

As Figures 7 and 8 show, standard deviation can vary quite substantially at the transition points.

Looking closely at Figure 8, the reader will note evidence of both divergence of linear response in October of each year (a large fluctuation building to the transition point every January) and then a critical slowing near the transition point, since the system needs time to return to a stable equilibrium. Note it is often May or June of each year before the system stabilizes, i.e., before the effect of the January transition point fully diminishes.

Since Figure 7 does not show the divergence of linear response and critical slowing as clearly as Figure 8 does, Table 2 is a list of the seasonal factors derived from an additive X-11 model[6] applied to the flows data.

As Table 2 shows we again have evidence of both divergence of linear response (large fluctuations near the transition point every January) and critical slowing (near the transition point, the system needing more time to return to stable equilibrium). Note here, as in the daily data, it is often May or June of each year before the system stabilizes, i.e., before the effect of the high January seasonal factor fully diminishes.

The only catastrophe flag we have not tested for is hysteresis, and we do this by comparing a logistic fit of the retail sales data to a catastrophe model fit. A logistic model cannot exhibit hysteresis [10], but it can exhibit arbitrarily fast acceleration.

[6] The X-11 model we use is the U.S. Bureau of the Census's X-11 variant of the Census Method II seasonal adjustment procedure available through Statistica®. The seasonal factors shown assume an additive model with adjustment for the number of business days in each month.

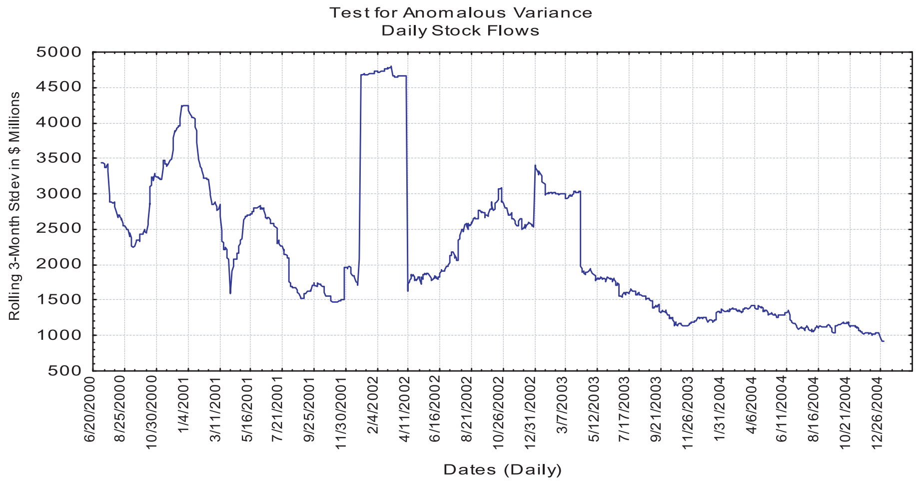


**Fig. 8.** Test for anomalous variance in daily U.S. stock fund flows data.

**Table 2.** Seasonal Factors for Monthly U.S. Stock Fund Flows Data.

| | Final seasonal factors: Monthly Stock Data | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Year | January | February | March | April | May | June | July | August | Septembr | October | November | December |
| 1984 | | | | | | | | | | | 239.820 | -435.922 |
| 1985 | 507.62 | 130.72 | -4.583 | 126.97 | -106.762 | 211.002 | -156.62 | -90.65 | -518.775 | 175.549 | 147.280 | -251.837 |
| 1986 | 423.40 | -8.70 | -56.978 | 97.13 | -29.089 | 147.351 | -53.64 | -29.09 | -370.401 | 71.241 | 173.119 | -108.188 |
| 1987 | 124.73 | -147.52 | -4.493 | 71.25 | 39.039 | 223.567 | -109.02 | -18.21 | -199.268 | -35.684 | 379.128 | 29.704 |
| 1988 | -235.83 | -231.97 | -29.303 | 49.09 | 118.835 | 59.071 | -7.59 | 48.21 | 13.055 | -161.122 | 616.094 | 210.977 |
| 1989 | -854.89 | -88.15 | -75.185 | -107.69 | 272.548 | 42.104 | -22.74 | 70.48 | 141.025 | -208.221 | 739.715 | 519.000 |
| 1990 | -1281.59 | -150.87 | -11.449 | -170.07 | 275.853 | -86.523 | 23.89 | 340.36 | -30.531 | -117.536 | 593.276 | 804.862 |
| 1991 | -1509.92 | -64.08 | 113.453 | -362.84 | 248.313 | 49.228 | -315.82 | 669.81 | -353.252 | 320.274 | 207.124 | 858.865 |
| 1992 | -1299.45 | -29.79 | 312.909 | -329.93 | -230.155 | 25.833 | -453.28 | 1140.11 | -548.449 | 511.098 | -325.715 | 931.788 |
| 1993 | -851.67 | -3.66 | 314.660 | -269.57 | -332.570 | -274.501 | -562.50 | 1363.93 | -414.538 | 462.589 | -587.191 | 750.187 |
| 1994 | 9.24 | -164.63 | 18.751 | -357.69 | -156.902 | -682.690 | -97.23 | 1059.96 | 89.264 | 206.025 | -763.581 | 307.226 |
| 1995 | 798.42 | -162.75 | -397.551 | -606.71 | 404.676 | -896.656 | 598.71 | 189.48 | 562.266 | -229.107 | -261.274 | -513.027 |
| 1996 | 1410.45 | -173.14 | -951.531 | -490.66 | 671.196 | -493.021 | 1349.43 | -981.06 | 748.614 | -521.440 | -57.657 | -863.354 |
| 1997 | 1604.24 | -432.28 | -586.553 | -252.49 | 490.886 | -168.465 | 1465.06 | -1483.14 | 742.852 | -457.697 | -251.705 | -643.842 |
| 1998 | 2025.05 | -1651.64 | -100.458 | 179.26 | 75.560 | 872.567 | 1027.99 | -1959.63 | 512.065 | -41.393 | -530.021 | -159.884 |
| 1999 | 2253.37 | -3065.17 | 858.500 | 573.32 | 65.621 | 830.783 | -33.41 | -1376.80 | -32.598 | 314.218 | 211.945 | 274.120 |
| 2000 | 2046.09 | -4178.13 | 781.716 | 1102.27 | 221.853 | 936.749 | -970.99 | -603.15 | -767.139 | 713.367 | 959.571 | 488.632 |
| 2001 | 1255.19 | -4075.73 | 916.719 | 1392.53 | -68.452 | 134.209 | -1505.73 | 355.27 | -729.475 | 744.905 | 1871.366 | 646.980 |
| 2002 | 130.01 | -3153.15 | 290.863 | 788.06 | -75.415 | 74.507 | -1113.89 | 237.49 | -331.002 | 556.889 | 2761.410 | 259.294 |
| 2003 | -1202.84 | -1371.73 | 70.532 | -58.72 | -188.116 | -534.655 | -355.75 | 118.98 | 208.828 | -26.632 | 3971.083 | -67.657 |
| 2004 | -2690.03 | 386.65 | -368.905 | -966.74 | -247.407 | -390.832 | 354.37 | -114.77 | 160.590 | -154.788 | 4591.190 | -364.567 |
| 2005 | -3687.41 | 1769.39 | -394.171 | -1418.72 | -881.628 | -300.207 | 749.62 | 32.73 | | | | |

## 4 Model construction and results

In our model-fitting routine we fit the three different models available in the Hartelman cuspfit[7] software: a linear model, a logistic model, and a cusp catastrophe model. Cuspfit uses log likelihood, AIC, and BIC as the main criteria to judge the better fit. Also, in order to take into account scale and position of the catastrophe process, a linear transformation of the data is implemented as in Wagenmakers et al. [10].

$$x \rightarrow \frac{x - \lambda}{\sigma}. \tag{4}$$

[7] Available at `http://users.fmg.uva.nl/hvandermaas`

For determining the shape of the cusp catastrophe model, the program estimates the control parameters $\alpha$ (the normal variable) and $\beta$ (the splitting variable) and the location and scale parameters $\lambda$ and $\sigma$.

As for the variables that make up $\alpha$ and $\beta$, i.e., $\alpha = xa0 + xa1 + xa2 + xa3 + xa4 + xa5$ and $\beta = xb0 + xb1 + xb2 + xb3 + xb4 + xb5$, these are the monthly or daily variables mentioned in Section 2 of this paper.

Hartelman's cuspfit program allows calculation of the fits on both an unconstrained and constrained basis. Unconstrained means all five variables enter on both the normal and splitting sides, while the constrained tests fix some of the parameters to zero for the normal variable, splitting variable, or both.

**Table 3.** Results of cuspfit runs on daily U.S. stock fund flows data.

| Model | Log Likelihood | AIC | BIC | No. of Parameters | Parameters Fixed at Zero | Normal Parameter(s) | Splitting Parameter(s) |
|---|---|---|---|---|---|---|---|
| Linear | –1979 | 2798.2 | 2804.5 | 5 | None | N/A | N/A |
| Cusp1 | $-5.7 \times 10^8$ | $-1.2 \times 10^8$ | $-1.2 \times 10^8$ | 10 | Up and down NYSE volume | Remaining daily variables | Remaining daily variables |
| Cusp2 | –57.3 | 100.2 | 103.7 | 10 | Gold fund flows, all 3 NYSE variables, bond fund flows, international stock fund flows | DJIA return prior month | Money market fund flows, bear fund flows |
| Logistic | –1560 | 2432.6 | 2445.8 | 6 | None | N/A | N/A |

**Table 4.** Results of cuspfit runs on monthly U.S. stock fund flows data.

| Model | Log Likelihood | AIC | BIC | No. of Parameters | Parameters Fixed at Zero | Normal Parameter(s) | Splitting Parameter(s) |
|---|---|---|---|---|---|---|---|
| Linear | –360.5 | 730.9 | 748.5 | 5 | None | N/A | N/A |
| Cusp1 | $-2.55 \times 10^9$ | $5.10 \times 10^9$ | $5.10 \times 10^9$ | 10 | Up and down NYSE volume | Remaining monthly variables | Remaining monthly variables |
| Cusp2 | –10.0 | 20.0 | 55.2 | 10 | Retail sales, NYSE volume, NYSE up volume bond fund flows | DJIA return prior month | Money market fund flows, down NYSE volume |
| Logistic | –322.0 | 655.5 | 676.6 | 6 | None | N/A | N/A |

Rather than showing all the possible fits for the daily and monthly data, we show the results for the unconstrained fit, i.e., all the variables appearing on both sides of the cusp equation, and then the best constrained fit.

As Table 3 shows, the best performing model for daily flows is the cusp model (Cusp2), with the DJIA returns lagged one day the normal variable and the current day's money market fund flows and the current day's bear fund flows as part of the splitting variable.

Table 4 shows similar results for the monthly data, with the exception that the NYSE down volume replaces bear fund flows in the splitting variable. It needs to be noted that bear fund flows are not a variable we can test in the monthly model, since these data have been collected only from 2000 onward. So, our hypothesis is that down volume in the monthly model is a proxy for a more-direct sentiment variable such as bear fund flows. We will discuss later tests that can be done to test for the potential proxy nature of stock market volume.

So, given the strong similarity of the two models, we discuss them together and note the differences between them as needed.

First, each of the catastrophe models is a very good fit to the data, much better than either the Cusp1 or the logistic models. We think the quality of the Cusp2 fit arises in part from a judicious choice of variables, which may cause a bifurcation. What we mean by this is that we found the DJIA returns lagged one month, and the money market and bear fund flows had the highest probability of generating a bifurcation according to Hartelman's kernel program.

We also knew from our monthly forecasting model the variables that had been influencing flows into and out of stock funds for the past several years. Prior to our use of ISCT, we had tested many variables to determine the ones that worked best in our forecasting model — useful in our catastrophe modeling process.

We also had the benefit of the work of Brown et al. and Goetzmann et al. to guide us to the variables that may work on a daily basis.

Finally, the stock fund flows data on both a monthly and daily basis were indeed i.i.d. We knew this from our calculation of the Hurst-Holder exponent and the Augmented Dickey-Fuller test. If the data had been stationary and not i.i.d., in particular if the data had shown some measure of long memory, we would have had a diffusion process, which could possibly be treated using a generalized Langevin equation but which would have lacked a Fokker-Planck representation (see Coffey et al. [34])[8].

Given the goodness of fit of Cusp2 in both cases, we have confirmation of the importance of sentiment variables to stock mutual fund flows. Only one of the variables is traditionally considered "objective"–the price, in our case the return on stocks. Volume variables are clear measures of sentiment as are bear fund flows. Flows in and out of money market funds, as demonstrated by both Goetzmann et al. [4] and Brown et al. [5], are also a sentiment variable.

As we noted above, our volume variables may be proxies for other variables such as economic news or volatility in the market. As is well known, periods of high volatility are often, though not always, accompanied by periods of increased selling of stock shares versus buying of the same. And the effect of news, especially bad economic news, often plays into what happens to prices and volume. So, our volume variables could be the manifestation of one or more latent variables, and we discuss ways of testing for this in the Conclusions section.

As for money market fund flows there is some discussion (see Goetzmann et al. [4] as an example) of the role money market fund flows play in determining stock fund flows. Three potential explanations are often given: (1) investors are using money funds as checking accounts, preliminary to investing in other assets; (2) investors are possibly following a common portfolio insurance strategy; or (3) money flows are reflecting investor sentiment concerning the equity premium.

Unlike Goetzmann et al. [4] we do not find sentiment about the equity premium to be the prevailing answer to the importance of money market fund flows because we do not find gold fund flows being a significant descriptor in our cusp models. Goetzmann et al. found a significant negative correlation between flows to stock funds and flows to gold funds, and this led them to conclude the movement in and out of money market funds into stock funds was primarily related to sentiment about the equity premium. It should be noted that Goetzmann et al. tested just 18 months of daily data, while we used almost five years of dailies in our computations. In the Conclusion sections we discuss the impact of this difference.

[8] Both Cobb and Hartelman in their development of SCT solve a Fokker-Planck equation, one using the Ito interpretation and the other the Stratonovich. Since no Fokker-Planck equation exists for processes with long memory, Coffey et al. outline some potential solutions involving the Klein-Kramers and other equations (pp. 668-669).

While we did find negative correlation between stock fund flows and gold fund flows as Goetzmann et al. did, when we included gold fund flows in our cusp calculations, none of these cusp models was as good a fit as Cusp2. The log likelihood, AIC values, and BIC values in the best model were worse than Cusp2 by a factor of five to ten. However, it is worth noting that including gold fund flows in our cusp model did result in a better model than either the logistic or linear model.

We think the role money market funds play in stock fund flows is quite possibly tripartite, i.e., it involves all three uses of money market funds mentioned in the paragraph above. We do not think there is only one answer to the question concerning the operation or use of money market funds and the flows in and out of stock funds. We think heterogeneity of money market fund flows *a la* Shefrin [7] plays a significant role on the sentiment side of the pricing kernel and some form of bi- or multi-modality is to be expected in this sentiment variable.

A surprising result to us was the insignificance of retail sales in the monthly model. In our one-step-ahead forecasting model mentioned above, we found lagged retail sales to be a significant descriptor of recent (the last five years) monthly stock fund flows.

It could be the prominence of retail sales in the last five years has been due more to the slow growth of real incomes over the period versus their faster growth in the years prior to 2000. For example, from 1996 through 1999 both retail sales and flows into stock funds grew at a good clip, with real income growth being about 4% per year. And, in the 15 years prior to 1996, income grew at a real annual rate of more than 2%. In comparison, in the past four years real income growth has been less than 1% per annum, so investors may be making the determination more often these days to either invest or consume. As a matter of fact, flows into U.S. stock funds over the past 12 months have been very close to the $10 billion per month estimated for the regular fund purchases made via 401(k), 403(b), and other similar plans. That is, individual investors are making their planned 401(k) et al. investments but do not appear to be placing any more money than that in mutual funds.

Finally, we note that we tested both our daily and monthly variables using a butterfly model versus a cusp model. We will not detail the construction of the butterfly model here, but instead refer readers to Rosser's [25] already-cited work. We note, however, the variables included in the best-fit butterfly models: normal variable — the return on the DJIA lagged one month and bear fund flows (bear fund flows appeared only in the daily model); splitting variable-money market fund flows; bias variable-bond fund flows, gold fund flows (in the daily model only), and down NYSE volume (in the monthly model only); and the butterfly variable-down NYSE volume (monthly model only), bond fund flows (in both models), gold fund flows, and bear fund flows (the last two in the daily model only).

In the best-fit butterfly model, while the butterfly model did marginally improve on the Cusp2 model in terms of fit — the AIC and BIC for the daily model went to 92.3 from 100.2, and 94.5 from 103.7, respectively — we do not have conclusive evidence that switching from a cusp to a butterfly model is strictly warranted.

Using a test of Wagenmakers and Farrell [33] we get a sense of how big or small the difference is between the two catastrophe models. The probability that the butterfly model is true and Cusp2 is not, given the BIC results, is:

$$\exp\left(-\frac{1}{2}\left(\Delta BIC_{butterfly-cusp}\right)\right) \Big/ \left[\exp\left(-\frac{1}{2}\left(BIC_{butterfly}\right)\right)+\exp\left(-\frac{1}{2}\left(BIC_{cusp2}\right)\right)\right] \quad (5)$$

A similar computation can be done using the AIC results.

The computation of the weighted BIC and weighted AIC results in the daily case favors the butterfly model. The results are more mixed in the monthly case — the weighted AIC favors the butterfly, while the weighted BIC favors the cusp.

We note the butterfly model conforms to Brown et al.'s and Goetzmann et al.'s findings on the importance of gold funds and bond funds in understanding daily stock flows. However, they did not test monthly data, and our monthly data cover a 20-plus year period. We suggest in the Conclusions section a way of confirming or falsifying the Cusp2 model for the daily periods.

## 5 Conclusions

By applying Hartelman's [7], Wagenmakers et al.'s [10,11], and others' recent work on invariant stochastic catastrophe theory, we have been able to demonstrate the importance of catastrophe theory for understanding the determinants of U.S. stock mutual fund flows.

The Cusp2 models provide better descriptions of the stock fund flows data than does a linear or logistic model. And while we have evidence that a butterfly model may better describe the daily data, the results for the monthly data are inconclusive, pointing to the need for more tests.

The Cusp2 models also confirm to a large extent the work of Goetzmann et al. [4] and Brown et al. [5] concerning the importance of sentiment on mutual fund net flows. And, given our success in using catastrophe models, we think the search for a priceable sentiment factor by both Brown et al. and Goetzmann et al. would benefit from a nonlinear approach. As Goetzmann et al. note, "In the analysis below, we find that behavioral factors are not captured by a linear combination of return portfolios."[9]

As we mentioned in the Introduction to this paper, we think sentiment variables enter into asset pricing models via the pricing kernel or stochastic discount factor, and as Shefrin [12] notes sentiment is part of that pricing kernel. We think the evolution of sentiment is best modeled using the SDEs related to elementary catastrophe theory as developed by Cobb, Hartelman, and others, as long as the process under study is stationary and i.i.d.

As a follow-up to our catastrophe models we suggest experimental economists or sociophysicists try to verify our cusp modeling results by: (1) testing the savings factor in an environment where a fixed portion of earnings is already committed to investment, e.g., 401(k) contributions, and then vary the growth of income and stock market returns to see if or when incremental purchases of securities start to kick-in; (2) attempting to find out the function of money market funds in terms of flows in and out of stock funds, i.e., are they primarily checking accounts, future equity premium views, portfolio insurance, or some combination of the three (Shefrin's multimodality assumption); (3) determining whether volume is a proxy for one or more (latent) variables such as volatility or is indeed a factor unto itself; and (4) determining if gold funds and bond funds play a significant role in the flows in or out of stock funds as the butterfly model implies. Evidence in favor of the daily butterfly model seems to indicate this. One way of improving on our results would be to lengthen the daily period looked at by both this author and Goetzmann et al.. Both have been working with recent daily data (the past five to seven years of market flows), so a longer historical record may help settle the question of cusp versus butterfly.

The author would like to thank Lipper's Don Cassidy, whose knowledge of mutual fund flows was of immense help. The author would also like to thank Dr. van der Maas for supplying a copy of Hartelman's dissertation and for making Hartelman's catastrophe theory tools available on his Web site. And thanks to DPR for all the encouragement he has given me during my studies.

## References

1. V.A. Walther, J. Fin. Econ. **39**, 209 (1995)
2. R.M. Edelen, J. Fin. Econ. **53**, 439 (1999)
3. D. Del Guercio, P.A. Tkac, J. Fin. Quant. Anal. **37**, 523 (2002)
4. W.N. Goetzmann, M. Masa, K.G. Rouwenhorst, Behavioral Factors in Mutual Fund Flows, *Yale ICF Working Paper No. 00-14*
5. S.J. Brown, W.N. Goetzmann, T. Hiraki, N. Shiraishi, M. Watanabe, Investor Sentiment in Japanese and U.S. Daily Mutual Fund Flows, *NBER, Working Paper No. 9470*
6. J.B. de Long, A. Shleifer, L.H. Summers, R.J. Waldmann, J. Finance **45**, 379 (1990)
7. P. Hartelman, *Stochastic Catastrophe Theory*, Ph.D. Dissertation, University of Amsterdam, 1997 (unpublished)
8. A. Ploeger, H.J. van der Maas, P.A.I. Hartelman, Psy. Bull. Rev. **9**, 26 (2002)
9. H.J. van der Maas, R. Kolstein, J. van der Plight, Soc. Methods Res. **32**, 125 (2003)
10. E.J. Wagenmakers, H.L.J. van der Maas, P.C.M. Molenaar, Catastrophe Theory, *Encyclopedia of Statistics in Behavioral Sciences*, edited by B. Everitt, D. Howel (Wiley, Chichester, 2005)

[9] Op. cit., p. 15.

11. E.J. Wagenmakers, P.C.M. Molenaar, P.A.I. Hartelman, H.L.J. van der Maas, Physica D, **211**, 263 (2005)
12. H. Shefrin, On Kernels and Sentiment, SSRN *Working Paper Series*
13. L. Cobb, Behavioral Science **23**, 360 (1978)
14. L. Cobb, Behavioral Science **26**, 75 (1981)
15. L. Cobb, P. Koppstein, N.H. Chen, J. Amer. Statist. Association **78**, 124 (1983)
16. L. Cobb, S. Zacks, J. Amer. Statist. Association **80**, 793 (1985)
17. L. Cobb, S. Zacks, Nonlinear Time Series Analysis for Dynamic Systems of Catastrophe Type, in *Nonlinear Time Series and Signal Processing*, edited by R.R. Mohler (Springer-Verlag, Berlin, 1988), pp. 97–118
18. D. Kwiatkowski, P. Phillips, P. Schmidt, Y. Shin, J. Econometrics **54**, 159 (1992)
19. S.J. Leybourne, B.P.M. McCabe, J. Business Economic Statistics **12**, 157 (1994)
20. R. Thom, *Structural Stability and Morphogenesis: An Outline of a General Theory of Models* (W.A. Benjamin, Reading, Mass., 1975)
21. E.C. Zeeman, *Catastrophe Theory — Selected Papers, 1972–1977* (Addison-Wesley, Reading, Mass., 1977)
22. R. Gilmore, *Catastrophe Theory for Scientists and Engineers* (Wiley, New York, 1981)
23. T. Poston, I. Stewart, *Catastrophe Theory and Its Applications* (Pitman, San Francisco, 1978)
24. E.C. Zeeman, On the Unstable Behavior of Stock Exchanges, in *Catastrophe Theory-Selected Papers, 1972–1977* (Addison-Wesley, Reading, Mass., 1977)
25. J.B. Rosser Jr., Chap. 5 of *From Catastrophe to Chaos: A General Theory of Economic Discontinuities* (Boston: Kluwer, 1991) (second edition, Vol. 1 same publisher, 2000)
26. J.B. Rosser Jr., The Rise and Fall of Catastrophe Theory in Economics: Was the Baby Thrown Out With the Bath Water?, `http://cob.jmu.edu/rosserjb/cat.rise.and.fall.doc`, September 2004
27. R.S. Zahler, H.J. Sussmann, Claims and Accomplishments of Applied Catastrophe Theory, *Nature*, October 27, 1977 (correspondence December 1 and December 29, 1977)
28. E. Roy Weintraub, Behavioral Science **28:1**, 79 (1983)
29. J. Honerkamp, *Stochastic Dynamical Systems* (VCH Publishers, New York, 1994)
30. L. Cobb, B. Watson, Math. Modeling **1**, 311 (1980)
31. B.W. Silverman, *Density Estimation for Statistics and Data Analysis* (Chapman and Hall/CRC, Boca Raton, Florida, 1986)
32. E. Parzen, Ann. Math. Statistics **33**, 1065 (1960)
33. E.J. Wagenmakers, S. Farrell, Psychonomic Bulletin Rev. **11**, 192 (2004)
34. W.T. Coffey, Y.P. Kalmykov, J.T. Waldron, *The Langevin Equation* (World Scientific, 2003)